NAG Toolbox for MATLAB

f08ue

1 Purpose

f08ue reduces a real symmetric-definite generalized eigenproblem $Az = \lambda Bz$ to the standard form $Cy = \lambda y$, where A and B are band matrices, A is a real symmetric matrix, and B has been factorized by f08uf.

2 Syntax

```
[ab, x, info] = f08ue(vect, uplo, ka, kb, ab, bb, 'n', n)
```

3 Description

To reduce the real symmetric-definite generalized eigenproblem $Az = \lambda Bz$ to the standard form $Cy = \lambda y$, where A, B and C are banded, f08ue must be preceded by a call to f08uf which computes the split Cholesky factorization of the positive-definite matrix B: $B = S^{\mathrm{T}}S$. The split Cholesky factorization, compared with the ordinary Cholesky factorization, allows the work to be approximately halved.

This function overwrites A with $C = X^{\mathrm{T}}AX$, where $X = S^{-1}Q$ and Q is a orthogonal matrix chosen (implicitly) to preserve the bandwidth of A. The function also has an option to allow the accumulation of X, and then, if z is an eigenvector of C, Xz is an eigenvector of the original system.

4 References

Crawford C R 1973 Reduction of a band-symmetric generalized eigenvalue problem *Comm. ACM* **16** 41–44

Kaufman L 1984 Banded eigenvalue solvers on vector machines *ACM Trans. Math. Software* **10** 73–86

5 Parameters

5.1 Compulsory Input Parameters

1: **vect – string**

Indicates whether X is to be returned.

vect = 'N'

X is not returned.

vect = 'V'

X is returned.

Constraint: **vect** = 'N' or 'V'.

2: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'

The upper triangular part of A is stored.

uplo = 'L'

The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **ka – int32 scalar**

If **uplo** = 'U', the number of superdiagonals, k_a, of the matrix A.

If **uplo** = 'L', the number of subdiagonals, k_a, of the matrix A.

Constraint: $\mathbf{ka} \geq 0$.

4: **kb – int32 scalar**

If **uplo** = 'U', the number of superdiagonals, k_b, of the matrix B.

If **uplo** = 'L', the number of subdiagonals, k_b, of the matrix B.

Constraint: $\mathbf{ka} \geq \mathbf{kb} \geq 0$.

5: **ab(ldab,∗) – double array**

The first dimension of the array **ab** must be at least $\mathbf{ka} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of the n by n symmetric band matrix A.

The matrix is stored in rows 1 to $k_a + 1$, more precisely,

> if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_a + 1 + i - j, j)$ for $\max(1j - k_a) \leq i \leq j$;
> if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(nj + k_a)$.

6: **bb(ldbb,∗) – double array**

The first dimension of the array **bb** must be at least $\mathbf{kb} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The banded split Cholesky factor of B as specified by **uplo**, **n** and **kb** and returned by f08uf.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab** The second dimension of the array **bb**.

n, the order of the matrices A and B.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldbb, ldx, work

5.4 Output Parameters

1: **ab(ldab,∗) – double array**

The first dimension of the array **ab** must be at least $\mathbf{ka} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

the upper or lower triangle of **ab** contains the corresponding upper or lower triangle of C as specified by **uplo**.

2: **x(ldx,*) – double array**

The first dimension, **ldx**, of the array **x** must satisfy

if **vect** = 'V', **ldx** $\geq \max(1, \mathbf{n})$;
if **vect** = 'N', **ldx** ≥ 1.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **vect** = 'V' and at least 1 if **vect** = 'N'

The n by n matrix $X = S^{-1}Q$, if **vect** = 'V'.

If **vect** = 'N', **x** is not referenced.

3: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **vect**, 2: **uplo**, 3: **n**, 4: **ka**, 5: **kb**, 6: **ab**, 7: **ldab**, 8: **bb**, 9: **ldbb**, 10: **x**, 11: **ldx**, 12: **work**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

Forming the reduced matrix C is a stable procedure. However it involves implicit multiplication by B^{-1}. When f08ue is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion.

8 Further Comments

The total number of floating-point operations is approximately $6n^2k_B$, when **vect** = 'N', assuming $n \gg k_A, k_B$; there are an additional $(3/2)n^3(k_B/k_A)$ operations when **vect** = 'V'.

The complex analogue of this function is f08us.

9 Example

```
vect = 'N';
uplo = 'L';
ka = int32(2);
kb = int32(1);
ab = [0.24, -0.11, -0.25, -0.03;
      0.39, 0.79, 0.48, 0;
      0.42, 0.63, 0, 0];
bb   =   [1.438749456993816,   1.050238722644967,   0.7462727898852248,
1.081665382639197;
      0.6602956445140701, -0.3885978477717261, -0.3050851079238761, 0];
[abOut, x, info] = f08ue(vect, uplo, ka, kb, ab, bb)
```

```
abOut =
    0.1159    0.3578    0.0330   -0.0256
    0.3299    1.0010    0.5842         0
    0.3912    0.7707         0         0
x =
```

```
     0
info =
           0
```